

**Performance Attribution White Paper**

**Introduction**

Performance attribution is a quantitative approach to analyzing the result of investment choices. Performance attribution is a tool used to evaluate portfolio managers that decomposes the account’s return vs. an appropriate benchmark. The analysis illustrates the decisions made by the portfolio manager.

Performance attribution determines how the portfolio manager’s asset allocation and selection of securities affects the portfolio’s performance when compared to a benchmark.

**Allocation and Selection Effect**

Total attribution is the difference between the portfolio’s return and the benchmark’s return. The two main components of performance attribution are allocation and selection.

Allocation measures the ability to effectively allocate assets to various sectors. It determines whether the overweighting or underweighting of sectors relative to a benchmark contributes negatively or positively to the account’s overall return. The allocation effect is illustrated by the chart below.

	Underperform	Outperform
Overweight	-	+
Underweight	+	-

Positive allocation occurs when the portfolio manager is overweight in a particular sector that has outperformed the benchmark and/or underweight in a sector that has underperformed the benchmark. On the other hand, negative allocation occurs when the portfolio manager is overweight in a sector that has done poorly and/or underweight in a sector that has done well.

Selection measures the ability to select securities within a sector relative to a benchmark. It determines the portion of performance attributable to the portfolio manager’s skill in selecting securities. When a portfolio manager is bullish on a security, he or she will be overweight in the security compared to the benchmark’s weight. Conversely, if a portfolio manager is bearish on a security, he or she will be underweight in the security compared to the benchmark’s weight. Positive selection occurs when the portfolio manager puts more weight than a benchmark in a security that performed well. On the contrary, negative selection occurs when the portfolio manager puts less weight than a benchmark in a security that performed well.



**Calculate Allocation Effect**

$$[(\text{Portfolio sector beginning weight plus purchases}) - (\text{Benchmark sector beginning weight})] \\ \times \\ [(\text{Benchmark sector return})]$$

**Example of Allocation Effect**

	Sector	Beginning Weight	Sector Return	Total Return
<b>Portfolio</b>	Basic Materials	10.00%	5.00%	7.50%
<b>S&amp;P 500</b>	Basic Materials	4.00%	2.25%	3.50%

$$[(10.00\%) - (4.00\%)] \times [2.25\%]$$

$$(6.00\%) \times (2.25\%) = 0.00135$$

In this example, the allocation effect is positive since the portfolio manager over weighted the basic materials sector, which performed better than the total benchmark return.

**Calculate Selection Effect**

$$[(\text{Benchmark sector beginning weight})] \times [(\text{Portfolio sector beginning weight plus purchases}) - (\text{Benchmark sector beginning weight})] \\ + \\ [(\text{Portfolio sector return}) - (\text{Benchmark sector return})] \times [(\text{Portfolio sector beginning weight plus purchases}) - (\text{Benchmark sector beginning weight})]$$

**Example of Selection Effect**

	Sector	Beginning Weight	Sector Return	Total Return
<b>Portfolio</b>	Basic Materials	10.00%	5.00%	7.50%
<b>S&amp;P 500</b>	Basic Materials	4.00%	2.25%	3.50%

$$[(4.00\%) \times (5.00\%) - (2.25\%)] + [[(10.00\%) - (4.00\%)] \times [(5.00\%) - (2.25\%)]]$$

$$(0.0011 + 0.00165) = 0.00275$$

In this example, the selection effect is positive since the portfolio manager selected securities that performed better than the securities held in the benchmark for that particular sector.



**Contribution to Return**

Contribution to return measures the contribution of certain portfolio constituents (symbols, sectors) to the portfolio's overall return. For example, contribution to return can decompose a portfolio's return to illustrate which sectors did or did not contribute positively to the portfolio's return.

**Calculate Contribution to Return**

$$[(\text{Portfolio sector beginning weight plus purchases})] \times [(\text{Portfolio sector return})]$$

**Example of Contribution to Return**

	Sector	Beginning Weight	Sector Return
<b>Portfolio</b>	Basic Materials	10.00%	5.00%

$[(10.00\%)] \times [(5.00\%)] = 0.5\%$

In the illustration above, 0.5% is the contribution to return for the basic materials sector.

**Frongello Method**

	Portfolio	S&P 500	Difference (Portfolio - S&P 500)	Allocation	Selection	Attribution
<b>Quarter 1</b>	4.50%	3.75%	0.75%	0.50%	0.25%	0.75%
<b>Quarter 2</b>	2.25%	3.25%	-1.00%	-0.25%	-0.75%	-1.00%
<b>Cumulative Return</b>	6.85125%	7.12188%	-0.27063%			

As illustrated in the table above, the sum of allocation and selection for quarter 1 and 2 does not equal the cumulative difference in return. Therefore, mathematical smoothing is used to make the total attribution effect equal the cumulative return. The method used is the Frongello.<sup>(1)</sup>



**Frongello Calculation**

$$F_{tb} = G_{tb} \prod_{j=1}^{t-1} (1 + R_j) + \bar{R}_t \sum_{j=1}^{t-1} F_{jb}$$

**Example of Frongello**

Adjusted Allocation

$$[(\text{Current quarter allocation})] \times [(\text{Previous quarter portfolio return})]$$

+

$$[(\text{Previous quarter allocation})] \times [(\text{Current quarter benchmark return})]$$

$$[(-0.25\% \times 1.045)] + [(0.50\% \times 3.25\%)]$$

$$(-0.26\%) + (1.63\%) = -0.2450\%$$

Adjusted Selection

$$[(\text{Current quarter selection})] \times [(\text{Previous quarter portfolio return})]$$

+

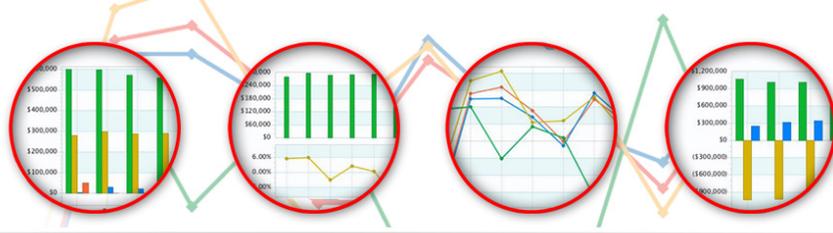
$$[(\text{Previous quarter selection})] \times [(\text{Current quarter benchmark return})]$$

$$[(-0.75\% \times 1.045)] + [(0.25\% \times 3.25\%)]$$

$$(-0.78\%) + (0.0081\%) = -0.775625\%$$

	Allocation	Selection	Adjusted Allocation	Adjusted Selection	Total
<b>Quarter 1</b>	0.50%	0.25%	0.50%	0.25%	
<b>Quarter 2</b>	-0.25%	-0.75%	-0.245%	-0.775625%	
<b>Total</b>			0.255000%	-0.525625%	-0.27063%

Using the Frongello method, the total adjusted attribution equals the difference of returns between the portfolio and the benchmark.



## Conclusion

In summary, attribution is the difference between the portfolio's return and the benchmark's return. The two main components of performance attribution are allocation and selection. You can use this analysis to determine the performance of the portfolio manager. If the attribution effect is positive, then the portfolio manager has contributed positively to the portfolio's overall return. In contrast, if the attribution effect is negative, then the portfolio manager has contributed negatively to the portfolio's overall return.

## End Notes

1. Andrew Scott Bay Frongello, CFA. Linking of Attribution Results. November 2005.
2. Calculations assume that purchases take place at the beginning of the day while sales, dividends and corporate actions take place at the end of the day.

## Legal Disclaimer

This report is for informational purposes only. The information provided is believed to be accurate, but the accuracy and completeness of the information is not guaranteed and Interactive Brokers has no liability with respect thereto. This report is intended only as a reference and should not be relied upon for the maintenance of books and records for tax, accounting, financial, regulatory reporting, or for any other purposes. Interactive Brokers does not provide proprietary research, recommendations or advice and is not responsible for any trading decisions resulting from or related to the information in this report.